Restricted structures on the Heisenberg algebra

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- Commutation relation: $p_j \circ q_k - q_k \circ p_j = -\mathbf{i}\hbar\delta_{j,k}$.
- Quickly, Weyl recognized a **representation** of a Lie algebra.

Heisenberg algebras

Definition

The (generalized) Heisenberg algebra $\mathcal{H}_{2n+1}$ *is the* $2n+1$ *dimensional vector space spanned by elements* $x_1, \cdots, x_n, y_1, \cdots, y_n, z$ *and equipped with a bilinear, antisymmetric operation* $[\cdot,\cdot]$ *defined by*

$$[x_j, y_j] = z, \ [x_j, y_k] = [x_j, x_k] = [x_j, z] = [y_j, z] = 0.$$

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Definition (Heisenberg, $n = 1$)

The three dimensional Heisenberg algebra is spanned by elements x, y, z and equipped with the bilinear, antisymmetric operation $[\cdot, \cdot]$ defined by

$$[x, y] = z, \; [x, z] = [y, z] = 0.$$

Lie algebras

Let $\mathbb{K}$ be an algebraic closed field of characteristic 0.

Definition

Let L be a $\mathbb{K}$ vector space. A **Lie bracket** *on L is a bilinear map $[\cdot,\cdot] : L \times L \longrightarrow L$ satisfying, for $x, y, z \in L$,*

1. $[x, x] = 0$ *(anticommutativity)*
2. $[x, [y, z]] + [y, [z, x]] + [z, [x, y]] = 0$ *(Jacobi identity).*

If L is endowed with such a bracket, we call the pair $(L, [\cdot,\cdot])$ a **Lie algebra**.

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$$[x, y] := \mu(x, y) - \mu(y, x)$$

is a Lie bracket on L called **commutator**.

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Important consequence: $M_n(\mathbb{K})$ endowed with the commutator is a Lie algebra.

Lie algebras

Definition

A linear map $\varphi : L_1 \longrightarrow L_2$ *is a* **Lie algebra map** *if*

$$\varphi\left([x,y]_1\right) = [\varphi(x), \varphi(y)]_2 \;\; \forall\, x, y \in L_1.$$

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Definition

A representation $\rho : L \longrightarrow End(V)$ *is a* **Lie algebra representation** *if* ρ *is a Lie algebra map, that is,*

$$\rho\left([x,y]\right) = \rho(x)\rho(y) - \rho(y)\rho(x).$$

Remark: One can also say that V is a L-module.

Lie algebras

Crucial example: the adjoint representation

$$\begin{aligned} \mathrm{ad} : L &\longrightarrow \mathrm{End}(L) \\ x &\longmapsto \mathrm{ad}_x : y \longmapsto [x, y]. \end{aligned}$$

Lie algebras

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- *Center of a Lie algebra L :*
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- *A Lie algebra L is called* **nilpotent of order n** *if, $\forall x_1, \cdots, x_n, y \in L$, we have*

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Example: The Heisenberg algebra is nilpotent of order 2.

Positive characteristic - restricted Lie algebras

Let $\mathbb{F}$ a field of characteristic $p > 0$ and A an associative $\mathbb{F}$-algebra. With the commutator, it's a Lie algebra. The adjoint representation is then given by

$$\mathrm{ad}_x(y) = xy - yx.$$

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Then, if $m = p$, we obtain

$$\mathrm{ad}_x^p(y) = x^p y - y x^p = \mathrm{ad}_{x^p}(y).$$

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Lemma

Let A be an associative algebra and let $a, b \in A$. Then,

$$(a+b)^p = a^p + b^p + \sum_{i=1}^{p-1} s_i(a,b),$$

with $is_i(a,b)$ being the coefficient of X^{i-1} in the polynomial expression $\mathrm{ad}_{aX+b}^{p-1}(a)$.

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$\rightsquigarrow$ it's much less friendly.

Positive characteristic - the p-mappings

The following definition is motivated by the previous example.

Definition (Jacobson)

A **restricted Lie algebra** *is a Lie algebra L equipped with a map $(\cdot)^{[p]} : L \longrightarrow L$ satisfying*

1. $(\lambda x)^{[p]} = \lambda^p x^{[p]}$, $x \in L$, $\lambda \in \mathbb{F}$;

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2. $\left[x, y^{[p]}\right] = [[\dots[x, \overbrace{y], y], \dots, y}^{p \text{ terms}}]$;
3. $$(x+y)^{[p]} = x^{[p]} + y^{[p]} + \sum_{i=1}^{p-1} s_i(x,y),$$

with $is_i(x,y)$ the coefficient of Z^{i-1} in $\mathrm{ad}^{p-1}_{Zx+y}(x)$. Such a map $(-)^{[p]} : L \longrightarrow L$ is called p-map. back

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1. Every associative algebra can be endowed with a restricted Lie algebra structure with the Frobenius map $x \longmapsto x^p$.

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3. Explicit expression for the sum of the s_i:

$$\sum_{i=1}^{p-1} s_i(x, y) = \sum_{\substack{x_i = x \text{ or } y \\ x_p = x, \ x_{p-1} = y}} \frac{1}{\sharp\{x\}} [x_1, [x_2, [\dots, [x_{p-1}, x_p]\dots],$$

Positive characteristic - the p-mappings

Examples:

1. Let A be an associative algebra and

$$\mathrm{Der}(A) = \{D : A \to A \text{ linear}, \ D(ab) = D(a)b + aD(b) \ \forall a, b \in A\}.$$

Then, $\mathrm{Der}(A)$ is a restricted Lie algebra with the commutator and the p-mapping $D \mapsto D^p$.

Positive characteristic - the p-mappings

Examples:

2. Restricted $\mathfrak{sl}_2(\mathbb{F})$ (char $\mathbb{F} > 2$):

$$\mathfrak{sl}_2(\mathbb{F}) = \mathrm{span}_{\mathbb{F}}\{X, Y, H\},$$

with brackets $[X, Y] = H,\ [H, X] = 2X,\ [H, Y] = -2Y.$

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with brackets $[X, Y] = H,\ [H, X] = 2X,\ [H, Y] = -2Y$.
Then,

$$X^{[p]} = Y^{[p]} = 0,\ H^{[p]} = 2^{p-1}H$$

is the **unique** p-structure on $\mathfrak{sl}_2(\mathbb{F})$.

Positive characteristic - the p-mappings

Some properties:

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- Suppose that $(\cdot)^{[p]_1}$ and $(\cdot)^{[p]_2}$ are p-maps on L. Then

$$x^{[p]_1} - x^{[p]_2} \in Z(L), \ \forall x \in L.$$

Consequence: if $Z(L) = 0$, there is at most one p-map on L.

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Theorem (Jacobson's Theorem)

Let L be a n-dimensional Lie algebra over a field $\mathbb{F}$ of characteristic p. Suppose that $(e_j)_{j\in\{1,\cdots,n\}}$ is a basis of L such that it exists $y_j \in L$, $\left(\mathrm{ad}_{e_j}\right)^p = \mathrm{ad}_{y_j}$. Then it exists exactly one p-map such that $e_j^{[p]} = y_j, \ \forall j = 1, \cdots, n$.

Positive characteristic - the p-mappings

Definition

Let $\left(L_1, [\cdot,\cdot]_1, (\cdot)^{[p]_1}\right)$ *and* $\left(L_2, [\cdot,\cdot]_2, (\cdot)^{[p]_2}\right)$ *be restricted Lie algebras. A* **restricted morphism** *(or* p-**morphism***)* $\varphi : L_1 \longrightarrow L_2$ *is a Lie morphism that satisfies* $\varphi\left(x^{[p]_1}\right) = \varphi(x)^{[p]_2}$.

Restricted Heisenberg, $p > 2$.

Let $\mathbb{F}$ be a field of characteristic $p > 2$.

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- $[u^{[p]}, v] = [u, [u, \cdots, [u, v] \cdots]] = 0$.

Consequence: the image of $(\cdot)^{[p]}$ lies in $Z(\mathcal{H}) = \mathrm{span}\{z\}$.

Restricted Heisenberg, $p > 2$.

Proposition

Any p-structure on $\mathcal{H}$ is given on the basis $\{x, y, z\}$ by

$$x^{[p]} = \theta(x)z,$$
$$y^{[p]} = \theta(y)z,$$
$$z^{[p]} = \theta(z)z,$$

with $\theta : \mathcal{H} \to \mathbb{F}$ a linear form on $\mathcal{H}$.

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with $\theta : \mathcal{H} \to \mathbb{F}$ a linear form on $\mathcal{H}$.

Notation. We will denote a restricted Heisenberg algebra by $(\mathcal{H}, \theta)$.

Remark. Let $u \in (\mathcal{H}, \theta)$, $u = \alpha x + \beta y + \gamma z$, $\alpha, \beta, \gamma \in \mathbb{F}$. Then

$$u^{[p]} = (\alpha^p \theta(x) + \beta^p \theta(y) + \gamma^p \theta(z))\, z.$$

Restricted Heisenberg, $p > 2$.

Lemma

Let $(\mathcal{H}, \theta)$ and $(\mathcal{H}, \theta')$ be two restricted Heisenberg algebras. Then, any Lie isomorphism $\phi : (\mathcal{H}, \theta) \to (\mathcal{H}, \theta')$ is of the form

$$\begin{cases} \phi(x) & = ax + by + cz \\ \phi(y) & = dx + ey + fz \\ \phi(z) & = (ae - bd)z, \quad ae - bd \neq 0, \end{cases} \tag{1}$$

with $a, b, c, d, e, f \in \mathbb{F}$. Moreover, ϕ is a restricted Lie isomorphism if and only if

$$\begin{cases} \theta(x)u & = a^p\theta'(x) + b^p\theta'(y) + c^p\theta'(z) \\ \theta(y)u & = d^p\theta'(x) + e^p\theta'(y) + f^p\theta'(z) \\ \theta(z)u & = u^p\theta'(z), \end{cases} \tag{2}$$

where $u := ae - bd \neq 0$.

Restricted Heisenberg, $p > 2$.

Theorem

There are three non-isomorphic restricted Heisenberg algebras, respectively given by the linear forms $\theta = 0$, $\theta = x^*$ *and* $\theta = z^*$.

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- $x^*(x) = 1,\ x^*(y) = x^*(z) = 0;$
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- $x^*(x) = 1,\ x^*(y) = x^*(z) = 0;$
- $z^*(z) = 1,\ z^*(x) = z^*(y) = 0;$
- $(\mathcal{H}, 0)$ and $(\mathcal{H}, x^*)$ are p-nilpotent.
- $(\mathcal{H}, z^*)$ is not.

Restricted Heisenberg, $p = 2$.

Now, let $\mathbb{F}$ be a field of characteristic 2.

- in that case, $\mathcal{H}$ is isomorphic to $\mathfrak{sl}_2(\mathbb{F})$.
- we have

$$(x+y)^{[2]} = x^{[2]} + y^{[2]} + z;$$
$$(x+z)^{[2]} = x^{[2]} + z^{[2]};$$
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- **Consequence:**

$$(ax+by+cz)^{[2]} = a^2x^{[2]} + b^2y^{[2]} + c^2z^{[2]} + abz, \;\; a, b, c \in \mathbb{F}.$$

Theorem

There are two non-isomorphic restricted Heisenberg algebras, respectively given by the linear forms $\theta = 0$ and $\theta = z^*$.

Generalized Heisenberg algebras

We recall the generalized Heisenberg algebra of dimension $2n+1$:

$$\mathcal{H}_{2n+1} = \mathrm{Span}\{x_1, \cdots, x_n, y_1, \cdots, y_n, z\},$$

with bracket given by

$$[x_i, x_j] = 0,\ [x_i, y_j] = 0\ (i \neq j),\ [x_i, y_i] = z,\ [x_i, z] = [y_i, z] = 0.$$

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Proposition

Any p-map on $\mathcal{H}_{2n+1}$ is given by

$$x_i^{[p]} = \theta(x_i)z, \ y_i^{[p]} = \theta(y_i)z, \ z^{[p]} = \theta(z)z,$$

with $\theta : \mathcal{H}_{2n+1} \to \mathbb{F}$ be a linear form.

Generalized Heisenberg algebras

We have the following results:

$$(\mathcal{H}, 0) \ncong (\mathcal{H}, x_i^*),\ (\mathcal{H}, 0) \ncong (\mathcal{H}, y_i^*),\ (\mathcal{H}, 0) \ncong (\mathcal{H}, z^*);$$

$$(\mathcal{H}, z^*) \ncong (\mathcal{H}, x_i^*),\ (\mathcal{H}, z^*) \ncong (\mathcal{H}, y_i^*).$$

Last Slide of the Day

Thank you for your attention!

Main reference: Q. Ehret & A. Makhlouf, *Deformations and Cohomology of restricted Lie-Rinehart algebras in positive characteristic*. (the preprint will be on arXiv asap)

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Question: événement à Mulhouse en juin??