Problems 7

1. An angle θ can be trisected if it is possible to construct (with ruler and compass) an angle of value $\dfrac{\theta}{3}$.
   (a) Show that π can be trisected.
   (b) Show that $\frac{\pi}{3}$ cannot be trisected.
   (c) Bonus : show that θ can be trisected if and only if the polynomial $4x^3 - 3x - \cos(\theta)$ is reducible over $\mathbb{Q}(\cos(\theta))$.

2. (from Recitation 6)
   (a) Let $\alpha_1, \alpha_2, \alpha_3$ denote the complex roots of $x^3 - 2$. Show that $[\mathbb{Q}(\alpha_1, \alpha_2) : \mathbb{Q}] = 6$.
   (b) Let $K = \mathbb{Q}(\sqrt{2}, i)$. Show that $[\mathbb{Q}(\sqrt{2}, i) : \mathbb{Q}] = 4$.

3. Let $K = \mathbb{Q}(\alpha)$, where α is a root of $x^3 - x - 1$. Determine the irreducible polynomial of $\gamma := 1 + \alpha^2$ over $\mathbb{Q}$.

4. Determine the irreducible polynomial for $\alpha = \sqrt{3} + \sqrt{5}$ over $\mathbb{Q}$, then over $\mathbb{Q}(\sqrt{5})$.