Problems 1

1. Let G be a group. Show that the inverse of an element is unique.
2. Let G be a group and let $a, b, c \in G$. Show that $ab = ac \Rightarrow b = c$.
3. For $n \geq 1$, consider
$$SL_n(\mathbb{C}) = \{M \in GL_n(\mathbb{C}),\ \det(M) = 1\}.$$
Show that $SL_n(\mathbb{C})$ is a subgroup of $GL_n(\mathbb{C})$.
4. Let $H = \{1, \pm i, \pm j, \pm k\}$ with the relations $i^2 = j^2 = k^2 = ijk = -1$. Show that H is a group, called quaternions group. Is it abelian ? Compute $(ij)^5$.
5. Let $\sigma = (134)(256) \in S_6$. Compute σ^3. Same question with $\tau = (123456)$.
6. For $n \geq 2$, Show that S_n is generated by the $n-1$ transpositions $(12), (13), \cdots, (1n)$.
7. Let λ, μ be real numbers. Define a binary operation $*$ on $\mathbb{R}$ by
$$a * b := \lambda ab + \mu(a + b).$$
Find conditions on λ, μ such that $*$ is associative.
8. (Later) Let $n \geq 3$ and consider $A_n = \{\sigma \in S_n,\ sgn(\sigma) = +1\}$. Show that A_n is generated by the 3-cycles.