

## Problems 4

1. Let  $G$  be the group  $\langle a \rangle \times \langle b \rangle$ , where  $|a| = 8$ ,  $|b| = 12$ . Let  $K = \langle (a^2, b^3) \rangle$ . Compute the order of  $\overline{(a^4, b)} \in G/K$ .
2. Show that  $\mathbb{Q}/\mathbb{Z}$  is an infinite abelian group in which every element has a finite order.
3. Let  $G$  be a group and  $H$  a subgroup. Recall that the *index* of  $H$  in  $G$ , denoted by  $[G : H]$ , is the number of left cosets of  $H$ . If there is a chain of inclusions of subgroups  $K \subset H \subset G$ , show that
$$[G : K] = [G : H][H : K].$$
4. Recall the circle group  $\mathbb{C}^0 = \{z \in \mathbb{C}, |z| = 1\}$ . Show that the group  $\mathbb{C}^\times/\mathbb{C}^0$  is isomorphic to  $(\mathbb{R}, +)$ .
5. (From HW2). Classify the groups of order 6 by considering the following cases :
  - (a) there is an element of order 6 ;
  - (b) there is an element of order 3 and no element of order 6 ;
  - (c) all elements have order 1 or 2.