Problems 7

1. Let $G = S_4$. Explain why G admits a subgroup of order 8 and exhibit one.
2. Same question for D_5 and orders 2 and 5.
3. Show that all reflexions are conjugated in D_n.
4. Let p prime and $1 \leq n < p$. Let G be a group of order pn and H a subgroup of order p. Show that H is normal.
5. Recall that a group G is called *simple* if its only normal subgroups are $\{1\}$ and G itself. Let G be a group of order 200. Show that G cannot be simple.
6. Let G be a group of order 10.
   (a) Show that there exists only one 5-Sylow $K =< x >$, with x some element of order 5.
   (b) Let H be a 2-Sylow. Explain why $H =< y >$, with y some element of order 2 and show that $K \cap H = \{1\}$.
   (c) Show that there exists $1 \leq r \leq 4$ such that $yx = x^r y$.
   (d) Show that elements of the form $x^i y^j$ are distinct and deduce the group law of G.
   (e) Show that r cannot be equal to $2, 3$.
   (f) Conclusion : show that $G \simeq C_5 \times C_2$ or $G \simeq D_5$.