NAME :

# Quiz 1

Let $n > 1$ and consider the set $\mathbb{Z}^n = \left\{ (x_1, \cdots, x_n), \ x_i \in \mathbb{Z} \right\}$ equipped with the composition law

$$(x_1, \cdots, x_n) * (y_1, \cdots, y_n) := (x_1 + y_1, \cdots, x_n + y_n).$$

1. Show that $(\mathbb{Z}^n, *)$ is a group.

2. Consider the map

$$\varphi : \mathbb{Z}^n \to \mathbb{Z}/2\,\mathbb{Z}, \quad (x_1, \cdots, x_n) \mapsto \sum_{i=1}^{n} x_i \text{ mod } 2.$$

(a) Show that $\varphi$ is an homomorphism of groups. Is it injective ? Surjective ?

(b) Show that $H = \left\{ (x_1, \cdots, x_n) \in \mathbb{Z}^n, \ \sum_{i=1}^{n} x_i \text{ is even} \right\}$ is a normal subgroup of $\mathbb{Z}^n$.

(c) In the case where $n = 3$, describe $\text{Ker}(\varphi)$.