NAME :

Quiz 2

We are working over $\mathbb{R}^n,\ n \geq 1$. Recall that the distance between two points $u, v \in \mathbb{R}^n$ is defined by the length $|u - v|$ of the vector $u - v$, where $|x| := \sqrt{x \cdot x}$, with $\cdot$ is the usual dot product of $\mathbb{R}^n$.

1. Recall the definitions of
   (a) an isometry of $\mathbb{R}^n$;
   (b) an orthogonal operator of $\mathbb{R}^n$.
2. Show that any orthogonal operator is an isometry.
3. Let $G_a = \{f : \mathbb{R} \to \mathbb{R},\ x \mapsto ax + b,\ (a, b) \in \mathbb{R}^* \times \mathbb{R}\}$ the group of affine transformations of $\mathbb{R}$.
   (a) Show that G_1 is a normal subgroup of G_a (no need to show that G_a is a group).
   (b) Show the isomorphism $G_a/G_1 \cong \mathbb{R}^*$.