

NAME :

Quiz 2

Exercise. Consider the field extension $\mathbb{Q}(\zeta_8)$, where ζ_8 is the primitive 8-th root of unity; namely,

$$\zeta_8 := \frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}.$$

1. Compute ζ_8^2 and ζ_8^4 .
2. Use Part (1) to find the irreducible polynomial of ζ_8 over \mathbb{Q} , then deduce $[\mathbb{Q}(\zeta_8) : \mathbb{Q}]$.
3. Use Part (2) to show that the cyclotomic field $\mathbb{Q}(\zeta_8)$ is isomorphic to $\mathbb{Q}(i, \sqrt{2})$.